The inhomogeneity of the atmosphere in which the oscillations take place leads to the fact that the lower part of the trajectory is traversed by the pendulum faster, and the upper part more slowly than in the case when the atmosphere is homogeneous. Figure 3 shows the dependence of the oscillation half-periods T_+ and T_- for the downward and upward deviations, on the inhomogeneity parameter δ (the dashed lines). Using these relations, or simply the dependence of the difference $\Delta T = T_- - T_+$ on δ, which differs little from the direct proportionality and depends weakly on ω_0, we can also determine δ using the measured value of the difference ΔT.

In all the motions discussed above, the reaction N of the line becomes equal to zero. In general, the oscillations are not planar.

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THE SOLUTIONS OF THE EQUATIONS OF MOTION OF THE KOVALEVSKAYA TOP IN FINITE FORM*

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Elementary transformations of phase variables are used to obtain several novel forms of the system of Euler-Poisson (EP) equations with Kovalevskaya conditions /1/. It is shown that the use of such equations makes possible not only the detection, but also the construction in a finite explicit form, of a solution for all four classes of degenerate motions mentioned by Appel'rot in /4/, and inadequately studied up to now, without using Kovalevskaya quadratures /2, 3/. In particular, an explicit solution is given in a novel form for the third class. The new forms of the equations of motion are used in a unique manner to study some particular results of investigation of degenerate solutions obtained by various methods /5-8/.

1. The initial equations. Using the Kovalevskaya conditions, we will write the EP equations and their algebraic first integrals in the form

$$2p^{\cdot} = qr, \quad 2q^{\cdot} = -rp - c_0\gamma'', \quad r^{\cdot} = c_0\gamma' \tag{1.1}$$

$$\gamma^{\cdot} = r\gamma' - q\gamma'', \quad \gamma'^{\cdot} = p\gamma'' - r\gamma, \quad \gamma''^{\cdot} = q\gamma - p\gamma'$$

$$2(p^2 + q^2) + r^2 - 2c_0\gamma = 6l_1, \quad 2(p\gamma + q\gamma') + r\gamma'' = 2l \tag{1.2}$$

$$\gamma^2 + \gamma'^2 + \gamma''^2 = 1, \quad (p^2 - q^2 + c_0\gamma')^2 + (2pq + c_0\gamma')^2 = k^2$$

where a dot denotes the time derivative. Let us introduce the complex variables

$$x_n = p + \varepsilon_n iq, \quad \xi_n = (p + \varepsilon_n iq)^2 + c_0(\gamma + \varepsilon_n i\gamma'), \quad n = 1, 2 \tag{1.3}$$

$$i = \sqrt{-1}, \quad \varepsilon_1 = 1, \quad \varepsilon_2 = -1$$

and rewrite (1.1) and (1.2) in the form

$$2\varepsilon_n ix_n^{\cdot} = rx_n + c_0\gamma'', \quad 2ir^{\cdot} = x_2^2 - x_1^2 + \xi_1 - \xi_2 \tag{1.4}$$

$$\varepsilon_n i\xi_n^{\cdot} = r\xi_n, \quad 2i\gamma''^{\cdot} = \xi_2 x_1 - \xi_1 x_2 + x_1x_2(x_1 - x_2)$$

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$$r^2 = 6l_1 - (x_1 + x_2)^2 + \xi_1 + \xi_2 \tag{1.5}$$
$$c_0 r\gamma'' = 2lc_0 + x_1x_2(x_1 + x_2) - x_2\xi_1 - x_1\xi_2$$
$$c_0^2\gamma''^2 = c_0^2 - k^2 - x_1^2x_2^2 + x_2^2\xi_1 + x_1^2\xi_2, \quad \xi_1\xi_2 = k^2$$

Eliminating the variables r, γ'' and using (1.5), we obtain the following equations which are important later:

$$-4x_n^{\cdot 2} = R(x_n) + (x_1 - x_2)^2\xi_n, \quad 4x_1 x_2^{\cdot} = R(x_1, x_2) \tag{1.6}$$

Here

$$R(x) = \sum_{\nu=0}^{4} A_\nu x^{4-\nu}, \quad R(x_1, x_2) = A_0x_1^2x_2^2 + A_2x_1x_2 +$$
$$^1/_2 A_3(x_1 + x_2) + A_4, \quad A_0 = -1, \quad A_1 = 0, \quad A_2 = 6l_1,$$
$$A_3 = 4lc_0, \quad A_4 = c_0^2 - k^2$$

2. The first form of the equations. Let us transform Eqs.(1.6). We introduce the following new variables:

$$y_n = -\frac{(x_n - a)}{M}, \quad \eta_n = \frac{k^2(x_n - a)^4\xi_n^{-1}}{M^2} \quad (M = (x_1 - a)(x_2 - a)) \tag{2.1}$$

where a is a constant. After this substitution, Eqs.(1.6) will take the form

$$-4y_n^{\cdot 2} = Q(y_n) + (y_1 - y_2)^2\eta_n, \quad 4y_1^{\cdot}y_2^{\cdot} = Q(y_1, y_2) \tag{2.2}$$
$$Q(y) = R(a)y^4 - R'(a)y^3 + {}^1/_2R''(a)y^2 + 4ay - 1 \quad (R' = dR(x)/dx)$$
$$Q(y_1, y_2) = R(a)y_1^2y_2^2 - {}^1/_2R'(a)y_1y_2(y_1 + y_2) +$$
$$^1/_2R''(a)y_1y_2 - a^2(y_1 - y_2)^2 + 2a(y_1 + y_2) - 1$$

We note that $\eta_1\eta_2 = \xi_1\xi_2 = k^2$. System (2.2) has the structure of the initial system (1.6). Taking this into account, we shall introduce another two variables z, γ_3, so that

$$2\varepsilon_n i y_n^{\cdot} = zy_n + \gamma_3 \quad (n = 1, 2) \tag{2.3}$$

Substituting these expressions for the derivatives $y_n^{\cdot}$ into Eqs.(2.2), we arrive at a system of three linear algebraic equations for the dual products. Solving this system we obtain

$$z^2 = R(a)(y_1 + y_2)^2 - R'(a)(y_1 + y_2) - {}^1/_2R''(a) + 2a^2 + \eta_1 + \eta_2 \tag{2.4}$$
$$z\gamma_3 = {}^1/_2R'(a)y_1y_2 - a^2(y_1 + y_2) + 2a - (\eta_2y_1 + \eta_1y_2)$$
$$\gamma_3^2 = R(a)y_1^2y_2^2 + 2a^2y_1y_2 - 1 + \eta_1y_2^2 + \eta_2y_1^2$$

On the other hand, the variables z, γ_3 can be expressed in an elementary manner in terms of the phase variables of the EP equations. Indeed, the derivatives of y_1, y_2, with respect to t defined by Eqs.(2.1) are, by virtue of the equations of motion (1.4), as follows:

$$-2\varepsilon_n i y_{1,2}^{\cdot} = (rx_{2,1} + c_0\gamma'')(x_{2,1} - a)^{-2}; \quad y_{1,2} = (y_1, y_2),$$
$$x_{2,1} = (x_1, x_2)$$

Equating this to (2.3) we find that

$$\gamma_3 = (c_0\gamma'' + ar)/M, \quad z = r + (x_1 + x_2 - 2a)\gamma_3 \tag{2.5}$$

Let us obtain the derivatives $\eta_n^{\cdot}$. Differentiating expressions (2.1) with respect to η_n, and taking into account the EP Eqs.(1.4), we find that

$$\varepsilon_n i\eta_n^{\cdot} = z\eta_n, \quad n = 1, 2 \tag{2.6}$$

Next we find the derivatives of z, γ_3. Here we find it convenient to use relations (2.4). Differentiating them term by term and taking into account the values of the derivatives $y_n^{\cdot}, \eta_n^{\cdot}$ (2.3) and (2.6), we obtain

$$2iz^{\cdot} = R(a)(y_1^2 - y_2^2) - {}^1/_2R'(a)(y_1 - y_2) + \eta_1 - \eta_2 \tag{2.7}$$
$$2i\gamma_3^{\cdot} = R(a)y_1y_2(y_2 - y_1) + a^2(y_2 - y_1) + \eta_2y_1 - \eta_1y_2$$

Let the polynomial $R(x)$ have a real root a. We note that in all degenerate cases /4/ determined by the conditions 1) $k = 0$; 2) $3l_1 \pm k = 2l^2$; 3) $R(x)$ has a multiple root, and the polynomial $R(x)$ has real roots. Then the variables $y_1, y_2, \eta_1, \eta_2, z, \gamma_3$ will satisfy the system of equations

$$2\varepsilon_n i y_n^{\cdot} = z y_n + \gamma_3, \quad 2iz^{\cdot} = -{}^1/_2 R'(a)(y_1 - y_2) + \eta_1 - \eta_2 \qquad (2.8)$$
$$\varepsilon_n i \eta_n^{\cdot} = z\eta_n, \quad 2i\gamma_3^{\cdot} = \eta_2 y_1 - \eta_1 y_2 - a^2 (y_1 - y_2)$$

which has the following first integrals:

$$z^2 = -R'(a)(y_1 + y_2) + \eta_1 + \eta_2 + {}^1/_2 R''(a) + 2a^2 \qquad (2.9)$$
$$z\gamma_3 = {}^1/_2 R'(a)\, y_1 y_2 - a^2 (y_1 + y_2) - (\eta_2 y_1 + \eta_1 y_2) + 2a$$
$$\gamma_3^2 = \eta_1 y_2^2 + \eta_2 y_1^2 + 2a^2 y_1 y_2 - 1, \quad \eta_1\eta_2 = k^2$$

We note that the EP variables $p, q, r, \gamma, \gamma', \gamma''$ are rational functions of the new variables. Let us assume that $R'(a) = 0$. Then the set of three equations

$$2iz^{\cdot} = \eta_1 - \eta_2, \quad \eta_1^{\cdot} = z\eta_1 - i\eta_2^{\cdot} = z\eta_2$$

will form a closed system whose solution can be easily found. Knowing the variables z, η_1, η_2 as functions of time, we can find /6, 7/ the remaining three variables y_1, y_2, γ_3. Another, more complicated method was used in /6, 7/ to obtain system (2.8) in a somewhat different form, and the latter cases used to construct the solution in question in explicit form. The polynomial $R(x)$ has a multiple root; therefore the solution describes the fourth class of the simplest motions (according to Appel'rot's classification /4/).

3. **The second form of the equations.** We see that the constant a appears in Eqs. (2.8) and in their integrals (2.9). We find that, in general, we can use certain linear transformations of the phase variables to obtain equations not containing this constant. Indeed, let $R(a) = 0$, but $R'(a) \neq 0$. We write

$$4p_n = R'(a)\, y_n + 2a^2, \quad 4\gamma_0 = R'(a)\,\gamma_3 - 2a^2 z \qquad (3.1)$$

After this substitution (2.8) and (2.9) will take the form

$$2\varepsilon_n p_n^{\cdot} i = z p_n + \gamma_0, \quad 2iz^{\cdot} = 2(p_2 - p_1) + \eta_1 - \eta_2 \qquad (3.2)$$
$$\varepsilon_n i \eta_n^{\cdot} = z\eta_n, \quad 2i\gamma_0^{\cdot} = \eta_2 p_1 - \eta_1 p_2$$
$$z^2 + 4(p_1 + p_2) - \eta_1 - \eta_2 = A_2, \quad \eta_1\eta_2 = k^2 \qquad (3.3)$$
$$\gamma_0 z + \eta_2 p_1 + \eta_1 p_2 - 2p_1 p_2 = -{}^1/_2 A_4$$
$$\gamma_0^2 = \eta_1 p_2^2 + \eta_2 p_1^2 + I, \quad 16I = 4A_2 A_4 - A_3^2$$

Let us write Eqs.(2.2) in another new form

$$-4p_n^{\cdot 2} = f(p_n) + (p_1 - p_2)^2 \eta_n, \quad 4p_1^{\cdot} p_2^{\cdot} = f(p_1, p_2) \qquad (3.4)$$
$$f(p) = -4p^3 + A_2 p^2 - A_4 p + I$$
$$f(p_1, p_2) = -2p_1 p_2 (p_1 + p_2) + A_2 p_1 p_2 - {}^1/_2 A_4 (p_1 + p_2) + I$$

We note that the function $f(p)$ can be transformed by linear substitution of the arguments $p = s + {}^1/_2 l_1$ into the function $S(s) = f(s + {}^1/_2 l_1) = 4s^3 - g_2 s - g_3$ where $g_2 = k^2 - c_0^2 + 3l_1^3$, $g_3 = l_1(k^2 - c_0^2 - l_1^2) + l^2 c_0^2$, which plays a significant part in the Kovalevskaya analysis. Let us write system (3.2) with integrals (3.3) in terms of real variables. Let

$$p_n = x + \varepsilon_n i y, \quad \eta_n = \alpha + \varepsilon_n i\beta$$

Then

$$2x^{\cdot} = zy, \quad 2y^{\cdot} = -zx - \gamma_0, \quad z^{\cdot} = \beta - 2y \qquad (3.5)$$
$$\alpha^{\cdot} = z\beta, \quad \beta^{\cdot} = -z\alpha, \quad \gamma_0^{\cdot} = \alpha y - \beta x$$
$$z^2 + 8x - 2\alpha = A_2, \quad 2(\alpha x + \beta y) + \gamma_0 z - 2(x^2 + y^2) = -{}^1/_2 A_4 \qquad (3.6)$$
$$\gamma_0^2 - 2\alpha(x^2 - y^2) - 4\beta xy = I, \quad \alpha^2 + \beta^2 = k^2$$

The equations of motion in the form (3.5) may be found useful in the study of the general solution, and of various special cases. For example, if $k = 0$ we have $\alpha = 0$, $\beta = 0$, $\gamma_0 = \mathrm{const}$ and the system (3.5) will be reduced to three equations

$$2x^{\cdot} = zy, \quad 2y^{\cdot} = -zx - \gamma_0, \quad z^{\cdot} = -2y$$

with the integrals

$$z^2 + 8x = A_2, \quad \gamma_0 z - 2(x^2 + y^2) = -{}^1/_2 A_4$$

The variable z is found from the equation $z^{\cdot 2} = 2\gamma_0 z + A_4 - 16^{-1}(A_2 - z^2)$. Knowing z, we can easily find x, y: $8x = A_2 - z^2$, $2y = -z^{\cdot}$. This represents a new form of solution in the Delon case.

4. The third form of the equations. We shall transform system (3.5), assuming that the new variables $\gamma_1, \gamma_2, s_1, s_2, u, v$ are connected with the old variables by the following relations:

$$\eta_n = u_n^2,\quad u_n = u + \varepsilon_n iv,\quad ks_1 = ux + vy - {}^1/_2 ku \tag{4.1}$$
$$ks_2 = -vx + uy - {}^1/_2 ku,\quad 2k\gamma_n = \gamma_0 + {}^1/_2\varepsilon_n z\quad (n = 1, 2)$$

The new variables satisfy the system of equations

$$s_1^{\cdot} = -v\gamma_1,\quad s_2^{\cdot} = -u\gamma_2,\quad u^{\cdot} = (\gamma_1 - \gamma_2)v \tag{4.2}$$
$$\gamma_1^{\cdot} = -vs_1,\quad \gamma_2^{\cdot} = us_2,\quad v^{\cdot} = (\gamma_2 - \gamma_1)u$$

with first integrals (σ_1, σ_2, I_4 are constants)

$$\gamma_1^2 - s_1^2 = \sigma_1,\quad \gamma_2^2 + s_2^2 = \sigma_2,\quad u^2 + v^2 = k \tag{4.3}$$
$$(u + 2s_1)^2 - (v + 2s_2)^2 - 2(\gamma_1 + \gamma_2)^2 = I_4$$
$$\sigma_n = {}^1/_4 k^{-2} f({}^1/_2\varepsilon_n k) = {}^1/_8 c_0^2 k^{-2}(3l_1 - \varepsilon_n k - 2l^2)$$
$$I_4 = 2Ik^{-2} = 3l_1 - c_0^2 k^{-2}(3l_1 - 2l^2)$$

Combining the integrals (4.3), we can obtain an integral in the form of the sum of three squares

$$(\gamma_1 + s_1 + 2u - 2\gamma_2)^2 + (\gamma_1 - s_1 - 2u - 2\gamma_2)^2 + 4(s_2 - v)^2 = I_5 \tag{4.4}$$
$$I_5 = 2I_4 + 6(\sigma_1 + 2\sigma_2 + k)$$

From (4.3) and (4.4) it follows that all new variables have an upper limit, as well as a lower limit.

A system of equations of motion in the form (4.2) can be used to elucidate the properties of the general solution and to construct sufficiently simple particular solutions. Let us consider, as an example, the case when the constants σ_1, σ_2 vanish. Let $\sigma_2 = 0$. Then $3l_1 + k - 2l^2 = 0$. The condition determines Appel'rot's solution /4/ which he calls the simplest motion of the second class. The solution can easily be constructed with help of Eqs.(4.2).

Indeed, if $\sigma_2 = 0$, then $s_2 \equiv \text{const} = 0$, $\gamma_2 = \text{const} = 0$. Therefore, we shall have $s_1^{\cdot} = -v\gamma_1$, $u^{\cdot} = v\gamma_1$, and this implies that $s_1 + u = b_0 = \text{const}$. Taking into account the fact that $\gamma_1^2 = s_1^2 + \sigma_1$, $v^2 = k - u^2$, we rapidly discover that $u^{\cdot 2} = (k - u^2)[\sigma_1 + (b_0 - u)^2]$. Knowing $u(t)$, we can calculate $_1, \gamma_1, v$ by elementary methods and thus complete the solution. We obtain a solution of the same type if $\sigma_1 = 0$ and we assume that $\gamma_1 - s_1 = 0$.

5. The third class of motions. Let a single restriction $\sigma_1 = 0$ hold. The restriction determines the so-called third class of simplest motions. We shall construct this solution using the system of equations of motion in the form (4.2). The system differs from one known earlier /7, 8/ not only in its derivation, but also in the form of the quadratures. If $\sigma_1 = (\gamma_1 - s_1)(\gamma_1 + s_1) = 0$, then one of the factors must be constantly equal to zero. We can limit ourselves, without loss of generality, to considering only a single version $\gamma_1 - s_1 = 0$.

Then the integral I_5 will yield

$$(\gamma_1 - \gamma_2 + u)^2 - 2(vs_2 - u\gamma_2) = {}^1/_2(3l_1 + k) = l^2 + k$$

Let us write

$$z_1 = \gamma_1 - \gamma_2 + u,\quad \beta_1 = -us_2 - v\gamma_2,\quad \alpha_1 = vs_2 - u\gamma_2 \tag{5.1}$$

We find that the above variables satisfy the closed system of equations

$$z_1^{\cdot} = \beta_1,\quad \beta_1^{\cdot} = -z_1\alpha_1,\quad \alpha_1^{\cdot} = z_1\beta_1 \tag{5.2}$$

with the integrals

$$z_1^2 - 2\alpha_1 = l^2 + k,\quad \alpha_1^2 + \beta_1^2 = k\sigma_2 = {}^1/_4 c_0^2 \tag{5.3}$$

System (5.1) is easily solved. The dependence of the variable z_1 on time can be found from the equation

$$z_1^{\cdot 2} = (c_0 + l^2 + k - z_1^2)(z_1^2 + c_0 - l^2 - k) \tag{5.4}$$

If $l^2 + k < c_0$, then $z_1 = \mu\,\mathrm{cn}\,\tau_1$, where $\tau_1^{\cdot} = \sqrt{{}^1/_2 c_0}$, $\mu = \sqrt{c_0^2 + l^2 + k}$ and the modulus of the elliptic function

$$\varkappa_1 = (c_0 + l^2 + k)/(2c_0)$$

If $l^2 + k > c_0$, then $z_1 = \mu \,\mathrm{dn}\, \tau_2$, where $\tau_2^{\cdot} = {}^1/_2\mu$, and the modulus $\varkappa_2 = 2c_0/(c_0 + l^2 + k)$.

If $l^2 + k = c_0$, then $z_1 = \tau^{\cdot}/\mathrm{ch}\,\tau$, $\tau^{\cdot} = \mathrm{const} = \sqrt{2c_0}$.

Knowing $z_1(t)$, we can find $\alpha_1(t)$, $\beta_1(t)$ and we shall assume these functions to be known. But a knowledge of these three variables is insufficient to determine all six unknowns, and we must therefore determine another variable. The integral I_4 reduces, for $\gamma_1 = s_1$, to the form

$$\gamma_1^2 + (s_2 + v)^2 - 2z_1\gamma_1 = B \tag{5.5}$$
$$B = {}^1/_2(I_4 + 2\sigma_2 - k) = (c_0^2 - 4kl^2)/(4k)$$

Let us assume that

$$q_n = \gamma_1 + \varepsilon_n i(s_2 + v) \quad (n = 1, 2) \tag{5.6}$$

are the required variables. Then we can write the above relation in the form

$$q_1q_2 - z_1(q_1 + q_2) = B \tag{5.7}$$

Let us determine the time derivatives of q_n. Taking into account the equation of motion (4.2), we obtain $q_{1,2}^{\cdot} = -\varepsilon_n i\gamma_1 u_{2,1}$. Multiplying these relations term by term and remembering that $u_1u_2 = k$, $2\gamma_1 = q_1 + q_2$, we arrive at the relation $4q_1^{\cdot}q_2^{\cdot} = k(q_1 + q_2)^2$. Let us write the variables sought in terms of $q_1, q_2, \alpha_1, \beta_1, z_1$. If we put $\nu_n = \alpha_1 + \varepsilon_n i\beta_1$, then

$$u_{1,2} = \frac{k(z_1 - q_{2,1})}{k + \nu_{2,1}}, \quad \gamma_2 + \varepsilon_n i s_2 = \frac{\nu_n(q_n - z_1)}{k + \nu_n} \quad (n = 1, 2) \tag{5.8}$$

Substituting the expressions for u_1, u_2, γ_1 into the equations $q_{1,2}^{\cdot} = -\varepsilon_n i\gamma_1 u_{2,1}$ and taking into account the finite relation (5.7), we obtain two independent complex equations

$$q_n^{\cdot} = \lambda_n(q_n^2 + B), \quad \lambda_n = \frac{\varepsilon_n ik}{2(k + \nu_n)} \tag{5.9}$$

The real variable $z_2 = i(q_1q_2 + B)/(q_1 - q_2)$ is governed, by virtue of (5.9), by the equation

$$z_2^{\cdot} = \lambda_0(z_2^2 - B) \tag{5.10}$$
$$\lambda_0 = i(\lambda_1 - \lambda_2) = \frac{k(\alpha_1 + k)}{8k\alpha_1 + c_0^2 + 4k^2} = \frac{l^2 + k - z_1^2}{8(z_1^2 + B)}$$

The form of the solution of this equation depends on the sign of the constant B:

$$B = b^2 > 0, \quad z_2 = b\,\mathrm{cth}\,\theta, \quad \theta^{\cdot} = -b\lambda_0 \tag{5.11}$$
$$B = -b^2 < 0, \quad z_2 = b\,\mathrm{ctg}\,\theta_1, \quad \theta_1^{\cdot} = -b\lambda_0$$
$$B = 0, \quad (z_2^{-1})^{\cdot} = -\lambda_0$$

Knowing z_2, we can find q_1, q_2 from the following finite equations:

$$q_1q_2 - z(q_1 + q_2) = B, \quad q_1q_2 + iz_2(q_1 - q_2) = -B \tag{5.12}$$

We note that from (5.8) and (5.7) there follows the equation $z_1^2 + B = \delta^2$, where $\delta^2 = (\nu_1 + k)(\nu_2 + k)k^{-1} > 0$. We shall write this relation connecting the variables z_1, δ, in the parametric form

$$B = b^2 > 0, \quad z_1 = b\,\mathrm{ctg}\,\psi, \quad \delta = b\,\mathrm{cosec}\,\psi \tag{5.13}$$
$$B = -b^2 < 0, \quad z_1 = b\,\mathrm{cth}\,\psi_1, \quad \delta = b\,\mathrm{cosech}\,\psi_1$$

Using relations (5.11) and (5.13), we can write q_n in the form

$$1)\ B = b^2 > 0, \quad q_n = \frac{b(\sin\psi + \varepsilon_n i\,\mathrm{sh}\,\theta)}{\mathrm{ch}\,\theta - \cos\psi}, \quad \theta^{\cdot} = -b\lambda_0$$

$$2)\ B = -b^2 < 0, \quad q_n = \frac{b(\mathrm{sh}\,\psi_1 + \varepsilon_n i\sin\theta_1)}{\mathrm{ch}\,\psi_1 - \cos\theta_1}, \quad \theta_1^{\cdot} = -b\lambda_0$$

$$3)\ B = 0, \quad \frac{2}{q_n} = \frac{1}{z_1} + \frac{\varepsilon_n i}{z_2}, \quad (z_2^{-1})^{\cdot} = -\lambda_0.$$

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ANALYTICAL CONSTRUCTION OF VISCOUS GAS FLOWS USING THE SEQUENCE OF LINEARIZED NAVIER - STOKES SYSTEMS*

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Solutions of the complete Navier-Stokes system are constructed in the form of special series for a viscous, heat conducting continuous compressible medium. The zeroth-order term of the series transmits some exact solution of the initial system (e.g. all parameters of the medium are constants). Further terms of the series are determined by recurrence methods in the course of solving the linearized Navier-Stokes system, homogeneous for the first term and inhomogeneous for all remaining terms. The representations obtained are used to obtain approximate solutions of some boundary value problems. The process of stabilizing **unidirectional flow between** two fixed walls with constant heat flux specified on them is discussed, and an analogue of Poiseuille flow is constructed.

1. We consider the system of Navier-Stokes equations /1/

$$\frac{\partial\rho}{\partial t}+\mathbf{V}\cdot\nabla\rho+\rho\operatorname{div}\mathbf{V}=0 \tag{1.1}$$

$$\rho\left(\frac{\partial\mathbf{V}}{\partial t}+\mathbf{V}\left\|\frac{\partial v_\alpha}{\partial x_\beta}\right\|^T\right)+\mathrm{Eu}_1c_1^2\nabla\rho+\mathrm{Eu}_2b_1\nabla T=$$

$$\frac{1}{\mathrm{Re}}\Big[(\operatorname{div}\mathbf{V})\left(\nabla\mu'-\frac{2}{3}\nabla\mu\right)+\nabla\mu\left(\left\|\frac{\partial v_\alpha}{\partial x_\beta}\right\|+\left\|\frac{\partial v_\alpha}{\partial x_\beta}\right\|^T\right)+$$

$$\left(\mu'+\frac{1}{3}\mu\right)\nabla(\operatorname{div}\mathbf{V})+\mu\Delta\mathbf{V}\Big]$$

$$\rho c_v\left(\frac{\partial T}{\partial t}+\mathbf{V}\cdot\nabla T\right)+\mathrm{Eu}_2\theta_1b_1T\operatorname{div}\mathbf{V}=$$

$$\frac{1}{\mathrm{Pr}_1\mathrm{Re}}(\varkappa\Delta T+\nabla\varkappa\cdot\nabla T)+\frac{\theta_1}{\mathrm{Re}}\Big\{\mu'(\operatorname{div}\mathbf{V})^2+$$

$$\frac{2}{3}\mu\left[\left(\frac{\partial v_1}{\partial x_1}-\frac{\partial v_2}{\partial x_2}\right)^2+\left(\frac{\partial v_1}{\partial x_1}-\frac{\partial v_3}{\partial x_3}\right)^2+\left(\frac{\partial v_2}{\partial x_2}-\frac{\partial v_3}{\partial x_3}\right)^2\right]+$$

$$\mu\left[\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)^2+\left(\frac{\partial v_1}{\partial x_3}+\frac{\partial v_3}{\partial x_1}\right)^2+\left(\frac{\partial v_2}{\partial x_3}+\frac{\partial v_3}{\partial x_2}\right)^2\right]\Big\}$$

$$\mathrm{Eu}_1=\frac{c_1^{*2}}{u_0^2},\quad \mathrm{Eu}_2=\frac{b_1^*T_0}{\rho_0u_0^2},\quad \mathrm{Re}=\frac{\rho_0u_0L}{\mu^*}$$

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